DEFINITE INTEGRALS

Part I

A Definite Integral represents the exact area under the curve between points 'a' and 'b'.

$$\int_a^b f(x)\,dx = [\,F(x)]_a^b = F(b) - F(a) \text{ is called the definite integral}$$

of f(x) between the limits a and b. where $\frac{d}{dx}(F(x)) = f(x)$

DERIVATIVE OF ANTIDERIVATIVE (LEIBNITZ'S RULE)

If h(x) & g(x) are differentiable functions of x then,

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt = f[h(x)].h'(x) - f[g(x)].\ g'(x)$$

DEFINITE INTEGRAL AS LIMIT OF A SUM

$$\int_a^b f(x)dx = \lim_{n\to\infty} h[f(a) + f(a+h) + f(a+2h) + \ldots + f(a + \overline{n-1}h)]$$

$$= \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh) \text{ where } h = \frac{b-a}{n}$$

WALLI'S FORMULA & REDUCTION FORMULA

$$\int_0^{\pi/2} \sin^n x.\cos^m x dx = \frac{[(n-1)(n-3)\ldots 1 \text{ or } 2]\ [(m-1)(m-3)\ldots 1 \text{ or } 2]}{(m+n)(m+n-2)(m+n-4)\ldots .1 \text{ or } 2]} K$$

Where $K = \frac{\pi}{2}$ if both 'm' and 'n' are even ($m, n \in N$); otherwise K = 1

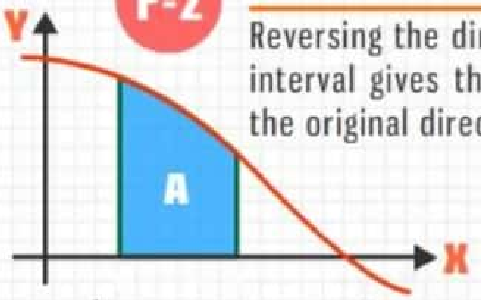
Part II

PROPERTIES OF DEFINITE INTEGRAL

P-1 **CHANGE OF VARIABLE:** $\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(r)dr$

P-2 **REVERSING THE INTERVAL**

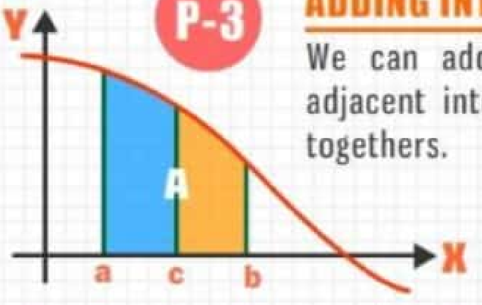
Reversing the direction of the interval gives the negative of the original direction.



$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

P-3 **ADDING INTERVAL**

We can add two adjacent intervals togethers.



$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

P-4

$$\int_{-a}^{a} f(x)dx = \int_0^a [f(x) + f(-x)]dx = \begin{cases} 0 & \text{if } f(x) \text{ is odd} \\ 2\int_0^a f(x)\,dx & \text{if } f(x) \text{ is even} \end{cases}$$

P-5

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx, \text{ in particular } \int_0^a f(x)dx = \int_0^a f(a-x)dx$$

P-6

$$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)\,dx \;; & \text{if } f(2a-x) = f(x) \\ 0 \;; & \text{if } f(2a-x) = -f(x) \end{cases}$$

P-7

$$\int_0^{nT} f(x)dx = n\int_0^T f(x)dx$$

where 'T' is the period of the function i.e. $f(x+T) = f(x)$

P-8

$$\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x)dx;$$ where $f(x)$ is periodic with period T & $n \in I$